

Chiral dynamics of the nuclear equation of state

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We present a new chiral power expansion scheme for the nuclear equation of state. The scheme is effective in the sense that it is constructed to work around nuclear saturation density. The leading and subleading terms are evaluated and are shown to provide an excellent equation of state. As a further application we considered the chiral quark condensate in nuclear matter. Already at nuclear saturation density we predict a substantially smaller reduction of the condensate as compared to conventional approaches.

1. Introduction

The nuclear equation of state lies at the heart of nuclear physics. It is therefore of considerable importance to find the appropriate form of chiral perturbation theory (χ PT), the most powerful tool of modern nuclear physics, and apply it systematically to the nuclear many body problem.

A key element of any microscopic theory for the nuclear equation of state is the elementary nucleon nucleon scattering process. In the context of χ PT this problem was first addressed by Weinberg who proposed to derive a chiral nucleon nucleon potential in time ordered perturbation theory [1]. The NN-phase shifts then follow from the solution of the Schroedinger equation appropriately fed with the chiral potential. This program was carried out by Ordonez, Ray and van Kolck [2]. Weinberg's scheme is plagued by two problems. First the systematic renormalization of the chiral potential scheme is an open problem and not resolved. The scheme is regularization scheme dependent. Second the use of a potential approach in nucleon nucleon scattering can be justified only if retardation effects are small.

To overcome both problems the present author proposed first to apply chiral power counting rules to the nucleon nucleon scattering amplitude directly [3]. This approach cures both problems. First the manifest covariant version of the chiral Lagrangian can be applied. A relativistic form of χ PT was applied before by Gasser [4] in the 1-nucleon sector but rejected and replaced by the heavy mass formulation of χ PT [5]. There are two problems inherent with the relativistic approach of [4]. First, any covariant derivative acting on the nucleon field produces the large nucleon mass, m_N , and therefore must be assigned the minimal chiral power zero. Thus an infinite tower of interaction terms need to be evaluated at a given finite chiral order. Second, the straightforward evaluation of relativistic diagrams involving nucleon propagators generates positive powers of the nucleon mass from loop momenta larger than the nucleon mass. The chiral power counting rules are spoiled.

The two problems are solved as follows. Rather than performing the $1/m_N$ expansion at the level of the Lagrangian density, as achieved in the heavy mass formulation of χ PT, it is equally well possible to work out this expansion explicitly at the level of any individual relativistic Feynman diagrams [3]. The second problem is overcome by a suitable reorganization of the infinite tower of chiral interaction terms. Of course we expect our relativistic scheme to be equivalent to the heavy fermion formulation of chiral perturbation theory in the one-nucleon sector.

The second important observation made first in [3] is the fact that the chiral power counting rules can be generalized for 2-nucleon reducible diagrams. For a given diagram each pair of intermediate nucleons causes a reduction of one chiral power as compared to the 'naive' chiral power. The non perturbative structures like the deuteron bound state are generated naturally since the bare 2-nucleon vertex is renormalized strongly so that it effectively carries chiral power minus one.

An attempt to apply the generalized chiral power counting rules of [3] to the nuclear many body problem quickly reveals that even though the pion dynamics remains perturbative the local 2-nucleon interaction requires extensive resummations.

2. Chiral expansion scheme for nuclear matter

It is straightforward to combine the chiral expansion of the nucleon scattering amplitudes with the low density expansion. Naive application to the energy per particle, $\bar{E}(k_F)$, of isospin symmetric nuclear matter would lead to an expansion of the form:

$$\bar{E}(k_F) = \sum_n \bar{E}_n \left(\frac{k_F}{m_\pi} \right) \left(\frac{k_F}{\Lambda} \right)^n \quad (1)$$

with $\Lambda \neq m_\pi$ and $\rho = 2k_F^3/(3\pi^2)$. The expansion coefficients \bar{E}_n are functions of the ratio k_F/m_π . Such a scheme is obviously restricted to extremely low density, if useful at all, since the typical scale Λ is small. This follows for example if one considers the 2-nucleon rescattering contribution to the 3-nucleon scattering process. Then the deuteron pole term induces structures like $k_F/\sqrt{m_N \epsilon_D}$ where $\epsilon_D \simeq 2$ MeV denotes the deuteron binding energy. One is lead to identify $\Lambda \sim \sqrt{m_N \epsilon_D}$.

A scheme working at nuclear saturation density therefore requires a further resummation. Obviously an expansion of the form

$$\bar{E}(k_F) = \sum_n \bar{E}_n \left(\frac{k_F}{m_\pi}, \frac{k_F}{\Lambda_S} \right) \left(\frac{k_F}{\Lambda_L} \right)^n \quad (2)$$

with typical small scales, Λ_S , like $\sqrt{m_N \epsilon_D}$ and typical large scales $\Lambda_L \simeq 4\pi f_\pi \simeq 1$ GeV must be achieved. The expansion coefficients \bar{E}_n are complicated and hitherto unknown functions of the Fermi momentum k_F . They can be computed in terms of the free space chiral Lagrangian furnished with a systematic resummation technique [6].

In this work we present a somewhat less microscopic approach in spirit close to the Brueckner scheme but more systematic in the sense of effective field theory. Since the typical small scale Λ_S is much smaller than the Fermi momentum $\bar{k}_F \simeq 265$ MeV at nuclear saturation density, one may expand the coefficient functions \bar{E}_n around \bar{k}_F in the

following manner

$$\bar{E}_n\left(\frac{k_F}{m_\pi}, \frac{k_F}{\Lambda_S}\right) = \bar{E}_n\left(\frac{k_F}{m_\pi}, \frac{\bar{k}_F}{\Lambda_S}\right) + \sum_{k=1}^{\infty} \bar{E}_n^{(k)}\left(\frac{k_F}{m_\pi}\right) \left(\frac{\Lambda_S}{k_F} - \frac{\Lambda_S}{\bar{k}_F}\right)^k. \quad (3)$$

Note that we do not expand in the ratio m_π/k_F . If one expanded also in this ratio m_π/k_F one would arrive at the Skyrme phenomenology [7] applied successfully to nuclear physics many years ago. It should be clear that this scheme is constructed to work around nuclear saturation density but will fail at small density. We note that also conventional approaches like the Walecka mean field or the Brueckner scheme are known to be incorrect at small density.

Technically our scheme can be generated by the effective Lagrangian density

$$\begin{aligned} \mathcal{L}_{int}(k_F) &= \frac{g_A}{2 f_\pi} \bar{N} \gamma_5 \gamma^\mu \cdot (\partial_\mu \vec{\pi}) \cdot \vec{\tau} N \\ &+ \frac{1}{8 f_\pi^2} \left(g_0(k_F) + \frac{1}{4} g_A^2 \right) \left(\bar{N} \gamma_5 \tau_2 C^{-1} \bar{N}^t \right) \left(N^t C \tau_2 \gamma_5 N \right) \\ &+ \frac{1}{8 f_\pi^2} \left(g_1(k_F) + \frac{1}{4} g_A^2 \right) \left(\bar{N} \gamma_\mu \vec{\tau} \tau_2 C^{-1} \bar{N}^t \right) \left(N^t C \tau_2 \vec{\tau} \gamma^\mu N \right) \end{aligned} \quad (4)$$

where the couplings $g_0 = g_0(k_F)$ and $g_1 = g_1(k_F)$ are density dependent. A more systematic derivation of the expansion (2) and (3) applying suitable resummation techniques will be presented elsewhere [6].

The chiral power counting rules are simplified significantly as compared to a fully microscopic scheme. The presence of a further small scale $k_F \sim Q \sim m_\pi$ does not anymore generate an infinite tower of diagrams, to be considered at a given chiral order, since by construction the troublesome local 2-nucleon vertex must not be iterated, i.e. terms proportional to $g_0^n(k_F)$ and $g_1^n(k_F)$ with $n > 1$ are already included in $g_0(k_F)$ and $g_1(k_F)$ and therefore must not be considered. The pion dynamics, if properly renormalized, remains perturbative like in the vacuum case. Consider for example the two loop diagrams depicted in Fig. 1 where the nucleon line with a 'cross' represents the projector onto the Fermi sphere

$$\Delta S_N(p) = (\gamma \cdot p + m_N) 2\pi i \Theta(p_0) \delta(p^2 - m_N^2) \Theta(k_F^2 - \vec{p}^2). \quad (5)$$

The first diagram in Fig. 1 is proportional to $g_{0,1}(k_F) k_F^6$ and is therefore ascribed the chiral order Q^6 since the effective vertex $g_{0,1}(k_F) \sim Q^0$ carries chiral power zero. The

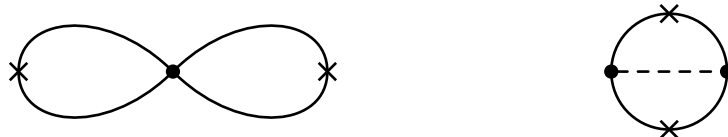


Figure 1. Leading contribution of chiral order Q^6 .

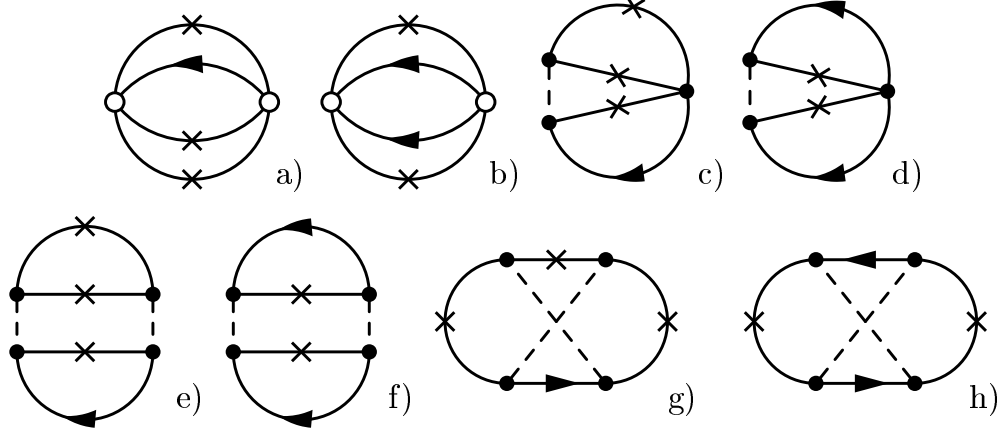


Figure 2. Subleading contribution of chiral order Q^7 .

second diagram, the one pion exchange contribution, is also of chiral order Q^6 since it is proportional to k_F^6 multiplied with some dimensionless function $f(m_\pi/k_F)$. In Fig. 2 we collected all diagrams of chiral order Q^7 . Here we introduced two types of 2-nucleon vertices. The filled circle represents the full vertex of (4) proportional to $g_{0,1} + g_A^2/4$ and the open circle the counter term proportional to $g_A^2/4$. The dashed line is the pion propagator and the directed solid line the free space nucleon propagator. We point out that the diagrams b), d), f) and h) in Fig. 2 are divergent. The leading chiral contribution of the sum of all diagrams, however, is finite.

It is instructive to also display the non relativistic form of the Lagrangian density (4) as suggested by Weinberg [1] in the context of the nucleon nucleon scattering problem

$$\begin{aligned}
 \mathcal{L}_{int}(k_F) &= \frac{g_A}{2f_\pi} \bar{N} (\vec{\sigma} \cdot \vec{\nabla}) (\vec{\pi} \cdot \vec{\tau}) N \\
 &+ \frac{1}{8f_\pi^2} \left(g_0(k_F) + \frac{1}{4} g_A^2 \right) (\bar{N} \vec{\sigma} \sigma_2 \tau_2 \bar{N}^t) (N^t \tau_2 \sigma_2 \vec{\sigma} N) \\
 &+ \frac{1}{8f_\pi^2} \left(g_1(k_F) + \frac{1}{4} g_A^2 \right) (\bar{N} \sigma_2 \vec{\tau} \tau_2 \bar{N}^t) (N^t \tau_2 \vec{\tau} \sigma_2 N)
 \end{aligned} \tag{6}$$

with N now a two component spinor field and $\bar{N} = N^\dagger$ for notational convenience. According to Weinberg the two particle reducible diagrams are to be evaluated with static pions. We therefore evaluate all diagrams of Fig. 2 with the interaction vertices of (6) and static pion propagators. The solid line with a 'cross' now represents the non relativistic limit of (5) and the directed solid line the free non relativistic nucleon propagator. We remind the reader that in the relativistic approach (4) we perform the chiral expansion at hand of a given Feynman diagram. For technical details we refer to [6]. Both schemes indeed lead to identical results for all diagrams except diagram h). Only in the extreme low density limit we find agreement of the non relativistic with the relativistic scheme also for diagram h). We conclude that it is incorrect to use static pions in two-particle

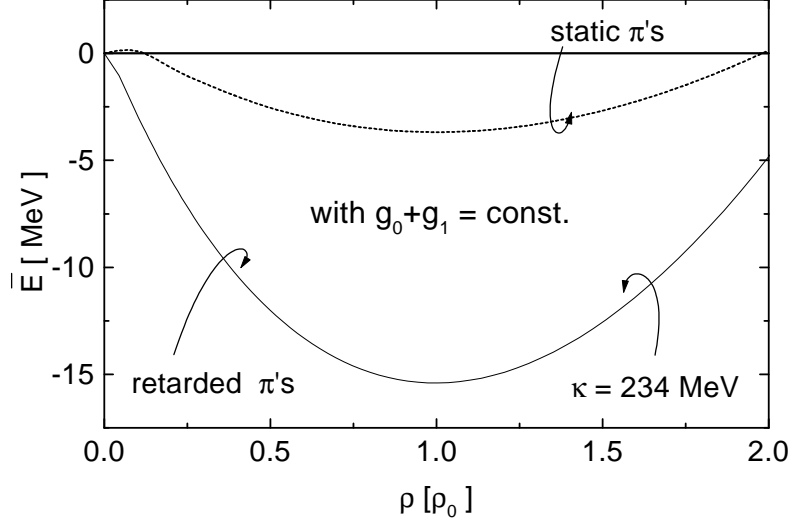


Figure 3. The equation of state for isospin symmetric nuclear matter.

reducible diagrams. Therefore the use of a potential in the χ PT approach of the nuclear many body problem *or* the nucleon nucleon scattering problem cannot be justified. A relativistic approach is required.

In Fig. 3 our result for the isospin symmetric nuclear equation of state is shown. We compare our relativistic approach (4) with the static potential approach of (6). The relevant coupling $g_0 + g_1$ is adjusted to obtain nuclear saturation at $k_F = 265$ MeV. The non relativistic scheme gives a rather poor equation of state as shown in Fig. 3. The binding energy and the incompressibility are too small. We emphasize that the coupling functions $g_{0,1}(k_F)$ are to be determined from the nuclear equation of state. It is therefore legitimate to cure the 'non relativistic' equation of state by giving the coupling a residual density dependence

$$g_0(k_F) + g_1(k_F) = \gamma_0 + \gamma_1 \frac{\bar{k}_F}{k_F} \left(1 - \frac{k_F}{\bar{k}_F}\right) + \gamma_2 \left(\frac{\bar{k}_F}{k_F}\right)^2 \left(1 - \frac{k_F}{\bar{k}_F}\right)^2 + \dots \quad (7)$$

However there is a strong consistency constraint: according to our scale argument (3) the density dependence of the couplings $g_{0,1}(k_F)$ must be weak. If the nuclear saturation required a strong density dependence our scheme had to be rejected. We turn to the relativistic scheme. Here the set of parameters $g_0 + g_1 \simeq 2.7$, $g_A \simeq 1.2$, $m_\pi \simeq 140$ MeV and $f_\pi \simeq 93$ MeV give an excellent result for the equation of state. The empirical saturation density $\bar{k}_F \simeq 265$ MeV and the empirical binding energy of 16 MeV are reproduced. The incompressibility with $\kappa \simeq 234$ MeV is also compatible with the empirical value (210 ± 30) MeV of [8]. We conclude that in fact the consistency constraint points towards the correct scheme (4).

3. The chiral order parameter $\langle \bar{q}q \rangle$

The quark condensate, $\langle \bar{q}q \rangle(\rho)$, is an object of utmost interest. It measures the degree of chiral symmetry restoration in nuclear matter. Furthermore it is an important input for QCD sum rules [9] or the Brown Rho scaling hypothesis [10]. According to the Feynman Hellman theorem the quark condensate can be extracted unambiguously from the total energy, $E(\rho)$, of nuclear matter once the current quark mass dependence of $E(\rho, m_Q)$ is known

$$\langle \bar{q}q \rangle(\rho) = \frac{1}{V} \frac{\partial}{\partial m_Q} E(\rho, m_Q) \quad (8)$$

with $m_Q = m_u = m_d$. Recall that $E(\rho)/V = (m_N + \bar{E}(\rho)) \rho$ is determined by the nuclear equation of state $\bar{E}(\rho)$. Since our chiral approach was set up to treat the pion dynamics and therewith the current quark mass dependence of the equation of state properly it is very much tailored to be applied to the quark condensate.

It is convenient to consider the relative change of the quark condensate since it is renormalization group invariant:

$$\frac{\langle \bar{q}q \rangle(\rho)}{\langle \bar{q}q \rangle(0)} = 1 - \frac{\Sigma_{\pi N} \rho}{m_\pi^2 f_\pi^2} - \frac{\alpha_\pi(\rho) \rho}{2 m_\pi f_\pi^2}. \quad (9)$$

The second term in (9) follows from the nucleon rest mass contribution to the total energy of nuclear matter together with the definition of the pion nucleon sigma term, $\Sigma_{\pi N}$:

$$\Sigma_{\pi N} = m_Q \langle N | \bar{q}q | N \rangle = m_Q \frac{d m_N}{d m_Q}. \quad (10)$$

The last term in (9) measures the sensitivity of the nuclear equation of state, $\bar{E}(\rho, m_\pi)$, on the pion mass

$$\alpha_\pi(\rho) = - \frac{2 m_\pi f_\pi^2}{\langle \bar{q}q \rangle(0)} \frac{\partial}{\partial m_Q} \bar{E}(\rho, m_\pi) = \left(1 + \mathcal{O}(m_\pi^2)\right) \frac{\partial}{\partial m_\pi} \bar{E}(\rho, m_\pi) \quad (11)$$

where we consider now the current quark mass $m_Q = m_Q(m_\pi)$ as a function of the physical pion mass. We emphasize that the second term in (9) written down first in [11–13] does not probe the nuclear many body problem and therefore should be considered with great caution. It is far from obvious that this term is the most important one at nuclear saturation density. The only term susceptible to nuclear dynamics is the last term in (9).

In Fig. 4 we present our result for the quark condensate in nuclear matter. We confront the 'leading' term driven by $\Sigma_{\pi N} \simeq 45$ MeV with first, our result applying static pions and second, our result including important retardation effects. The former result is rather close to the 'leading' order result. The nuclear many body dynamics as induced by static pions appear to effect the quark condensate little. This confirms results obtained in the Brueckner [14] and Dirac-Brueckner [15] approach. Note that neither the Brueckner nor the Dirac-Brueckner approach includes pionic retardation effects systematically. We celebrate the central result of this section: as shown in Fig. 4 the inclusion of pionic retardation effects leads to a significantly less reduced quark condensate in nuclear matter.

Our result implies strong consequences for the QCD sum rule approach to the properties of hadrons in nuclear matter.

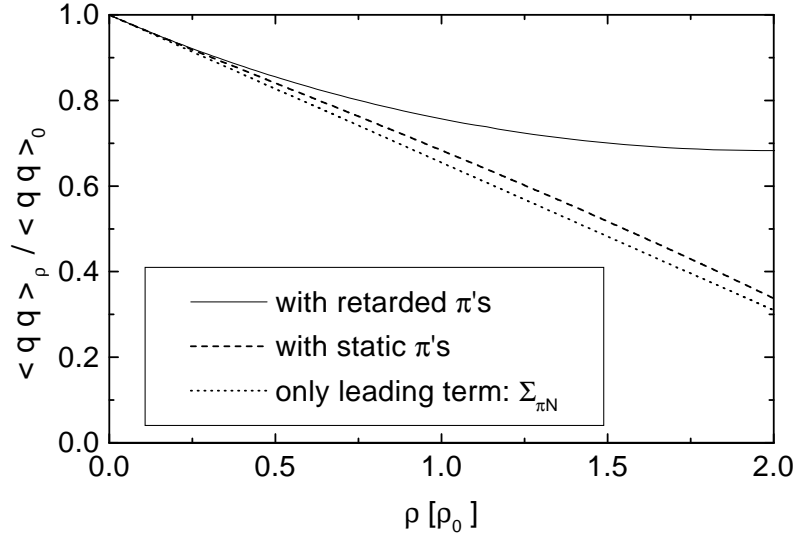


Figure 4. The quark condensate in isospin symmetric nuclear matter.

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